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Pulsations of the mass flow rate during pressure relief ☆

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Abstract

During two-phase flow blowdown from pressure vessels considerable pulsations of the discharged mass flow rate were found. Regions of mass flow rate instability were predicted by a linear stability analysis. The pulsations are caused by the following closed feedback circuit: boil up – level swell – void fraction of the discharged mixture – critical discharge rate – velocity of pressure decrease – boil up. They were also observed in transient simulations of the depressurization. Finally, the regions of instability were confirmed by experiments. The possibility of the occurrence of pulsations increases with the volume of the ventline and the volume void fraction of the discharged mixture. They may influence the rate of pressure relief from pressure vessels as well as from chemical reactors.
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Keywords: Pressure relief; Two-phase flow; Pulsations of the mass flow rate; Analysis of stability

1. Introduction

Emergency relief systems are usually very complex facilities regarding the geometries as well as the thermofluiddynamic processes within the system in case of pressure relief. For this reason feedback between the flow within the ventline and the processes within the vessel may effect the capacity of the entire system or parts of it. This paper reports on the results of an analysis of a special feedback between the level swell within the reactor vessel and the discharged volume flow rate. In case of two-phase flow venting this feedback may cause pulsations of the mass flow rate under certain conditions [1,2]. By a linear stability analysis regions of instability of the mass flow rate were predicted. The oscillations were also found at calculational results from codes developed for the transient simulation of the blowdown process. Finally the analytically predicted oscillations were confirmed by experiments.

The component under investigation is a pressure vessel, which is equipped with a vertical ventline connected at the top (Fig. 1). The relief device is located at the upper end of the ventline. It has a distinctly smaller cross section than the pipe. Therefore, it can be assumed that the critical flow conditions are always reached in the relief device if the vessel pressure is sufficiently high.

The sequence of the oscillations is shown in Fig. 1. Four stages are distinguished. After starting the relief the pressure decreases. Evaporation or degassing of the liquid inventory of the vessel leading to a two-phase mixture is the consequence. This leads to a level swell. In stage 1 the top level of the mixture is below the inlet of the ventline, but it is assumed that it still rises up. In stage 2 the level reaches the ventline and a two-phase mixture enters the ventline. Because of the assumption that the critical diameter is located at the relief device, the discharged volume flow rate is determined by the flow conditions at this location. In stage 2 still only gas flows through the relief device, with the consequence that the volume flow rate remains high. In stage 3 the mixture arrives at the relief device after a certain delay (time of transport). The critical discharge volume flow rate strongly depends on the local mass flow quality. The decrease of the mass flow quality at the critical cross section results in a decrease of volume release, which is connected with a decrease of the velocity of depressurization. This leads to a reduction of the gas production in the vessel. Under certain conditions the mixture level then starts to fall. In stage 4 the top level of the mixture is located again below the inlet of the ventline and only gas enters the ventline. In this stage still two-phase mixture is discharged. That means the volume flow rate remains low. Only after all the liquid is discharged from the ventline, the velocity

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Nomenclature

A	area	m^2
K	factor of the formula for the height of the froth region	
T	temperature	K
V	volume	m^3
a	auxiliary variable defined by Eqs. (B.10), (10)	
b	auxiliary variable defined by Eq. (B.11)	
c_p	specific heat capacity	$W \cdot kg^{-1} \cdot K^{-1}$
d	diameter	m
f	density of the interphase boundary surface	m^{-1}
g	constant of gravity acceleration	$9.81\ m \cdot s^{-2}$
h	specific enthalpy	$J \cdot kg^{-1}$
j	superficial velocity	$m \cdot s^{-1}$
k	prompt transfer function	
$\dot{m}$	mass flow rate	$kg \cdot s^{-1}$
p	pressure	Pa
q	heat flow density	$W \cdot m^{-3}$
r	specific heat of evaporation	$J \cdot kg^{-1}$
s	independent variable of the Laplacian space	s^{-1}
t	time	s
u	specific internal energy	$J \cdot kg^{-1}$
v	specific volume	$m^3 \cdot kg^{-1}$
w	transfer function in Laplacian space	
x	vapor mass fraction	
z	height	m

Greek symbols

Γ	density of the mass transfer rate	$kg \cdot m^{-3} \cdot s^{-1}$
Θ	jump function	
α	heat transfer coefficient	$W \cdot m^{-2} \cdot K^{-1}$
γ	auxiliary variable defined by Eq. (12)	m
ε	void fraction	
ζ	velocity	$m \cdot s^{-1}$
κ	adiabatic coefficient	
μ	factor of the discharge model considering the contraction of the flow	
ν	volume source related on vessel volume	s^{-1}
ν	volume source caused by evaporation related on the liquid volume	s^{-1}
ρ	density	$kg \cdot m^{-3}$
σ	surface tension	$N \cdot m^{-1}$
ϕ	auxiliary factor of the discharge model, defined by Eq. (3)	
ω	angular frequency	s^{-1}

Indices

comp	compression
crit	refers the values for critical flow
eva	evaporation
froth	froth
i	refers the phase (l or v)
in	values at the inlet of the ventline
delay	refers the delay time within the ventline
l	values of the liquid phase
level	values at the top level of the two-phase mixture
open	refers the transfer function of the open system
orif	values at the orifice
out	values at the end of the ventline = stagnation point at the discharge channel
phases	refers heat transfer within the single phases
m	refers a transfer function defined at Eq. (4)
$m\nu$	refers a transfer function defined at Eq. (5)
p	refers a transfer function defined at Eq. (9)
s	saturation
sep	refers a transfer function defined at Eq. (11)
sources	refers the heat transfer from external sources (wall)
v	values of the vapour phase
ventline	refers the ventline
vessel	values of the whole vessel
$\varepsilon\nu$	refers a transfer function defined at Eq. (6)

of depressurization increases again causing a re-established level swell [3].

At the top of a two-phase mixture a more or less developed frothy layer can always be found due to phase separation. It prevents a sudden change between one-phase and two-phase discharge. Therefore, continuous change of the mass flow quality of the two-phase mixture, which enters the ventline in case of small level fluctuations is more likely. In this case oscillations caused by the mechanism described above are also possible. The only difference is, that instead of a sudden change between only gas venting and two-phase venting an increase or decrease of the gas volume fraction occurs. The height of the froth (or transition) zone has a remarkable influence on the oscillations. The higher this zone is, the more the oscillations are attenuated.

A linear stability analysis was used in order to determine conditions for which the oscillations are possible.

2. Analysis of stability

The method of linear stability analysis is applied to the signals of the measured mass discharge for the considered system. An evaporating fluid is assumed. A linearisation of models for the calculation of the mass flow rate is carried out for the neighbourhood of an assumed working point. This working point is an imaginary state of the system, for that all the properties and their derivations are kept constant. It represents the system at a given time during the transient process. For the considered system the working point is mainly characterized by a consistent solution for the void

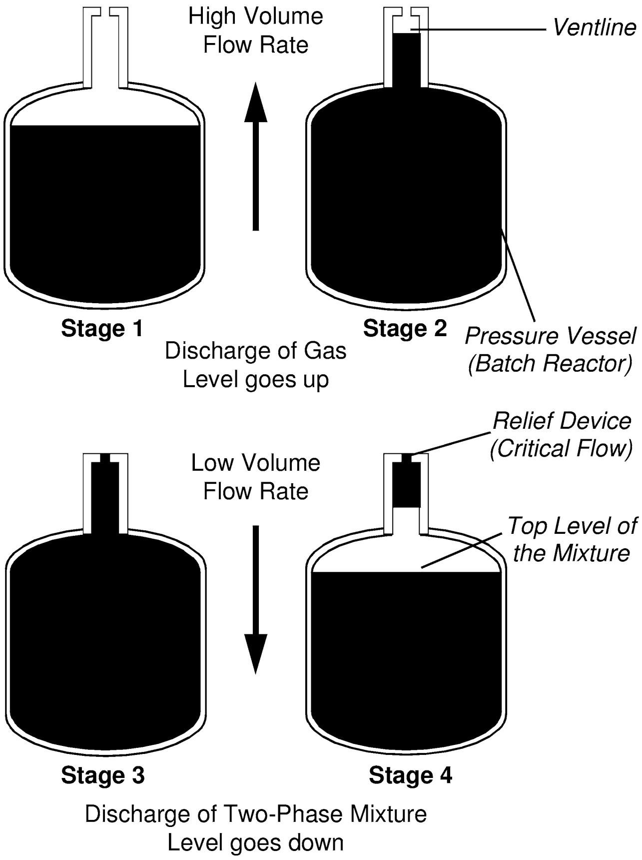

Fig. 1. Mechanism of the periodic oscillations.

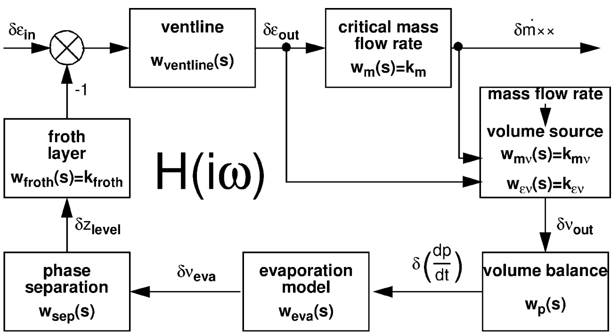

Fig. 2. Scheme of the transfer functions.

profile within the vessel, the void fraction in the ventline, the discharge flow rate, the velocity of depressurization and the evaporation rate. A small disturbance of one parameter cause disturbances of all the parameters. The transfer of such small disturbances is investigated. Fig. 2 shows the scheme for the transfer of the considered model variables. The modelling of the transfer of the disturbances bases on transfer functions formulated in the Laplacian space. The model is valid for an evaporating one-component system and a cylindrical pressure vessel.

An initial disturbance for the void fraction is assumed at the inlet of the ventline $\delta\varepsilon_{\mathrm{in}}$. The ventline was approximated by assuming a linear all-pass behaviour with a given delay time. That means that the shape and amplitude of the disturbance do not change within the ventline and the slip between the phases within the ventline is neglected. This assumption is reflected by the transfer function

$$w_{\mathrm{ventline}}(s) = \exp(-t_{\mathrm{delay}} s) \tag{1}$$

where t_{delay} is the delay time and s is independent variable of the Laplacean space. The delay time is calculated from the discharged volume flow rate and the volume of the ventline

$$t_{\mathrm{delay}} = \frac{(\rho^l(1-\varepsilon_{\mathrm{out}}) + \rho^v \varepsilon_{\mathrm{out}})}{\dot{m}} V_{\mathrm{ventline}} \tag{2}$$

When the disturbance of the void fraction reaches the end of the ventline, where the relief device is located, it causes a disturbance of the critical mass flow rate. It is assumed that a change of the mass flow quality at the relief device results in a sudden change of the flow rate, i.e., the inertia of the fluid in the discharge channel is neglected. For this reason the transfer function $w_m(s)$ is reduced to a transfer factor k_m. In general, the mass flow rate is a function of pressure and void fraction at of the relief device. The analyses can be performed for any chosen critical mass flow model. For the presented results a simple frozen flow model, derived in Appendix A, was used:

$$\dot{m} = \mu\phi A_{\mathrm{orif}} \sqrt{p\left(\rho^l(1-\varepsilon_{\mathrm{out}}) + \rho^v \varepsilon_{\mathrm{out}}\right)}$$
$$\text{with } \phi = \sqrt{2} - 1.0484\varepsilon_{\mathrm{out}}^{0.438} + 0.30603\varepsilon_{\mathrm{out}} \tag{3}$$

From this equations results:

$$\begin{aligned} w_m &= k_m = \frac{\partial}{\partial\varepsilon_{\mathrm{out}}}\dot{m} \\ &= \mu A_{\mathrm{orif}}\Bigg[\phi \frac{\rho^v - \rho^l}{2}\sqrt{\frac{\rho}{\rho^l(1-\varepsilon_{\mathrm{out}}) + \rho^v\varepsilon_{\mathrm{out}}}} \\ &\quad + \left(0.30603 - 0.4592\varepsilon_{\mathrm{out}}^{-0.562}\right) \\ &\quad \times \sqrt{p\left(\rho^l(1-\varepsilon_{\mathrm{out}}) + \rho^v\varepsilon_{\mathrm{out}}\right)}\Bigg] \end{aligned} \tag{4}$$

The disturbance of the discharged mass flow rate $\delta\dot{m}$ may be converted into the disturbance of the volume flow rate related on the vessel volume $\delta\nu_{\mathrm{out}}$ by the transfer factors $k_{m\nu}$ and $k_{\varepsilon\nu}$.

$$k_{m\nu} = \frac{\partial\nu_{\mathrm{out}}}{\partial\dot{m}} = -\frac{1}{V_{\mathrm{vessel}}(\rho^l(1-\varepsilon_{\mathrm{out}}) + \rho^v\varepsilon_{\mathrm{out}})} \tag{5}$$

$$k_{\varepsilon\nu} = \frac{\partial\nu_{\mathrm{out}}}{\partial\varepsilon_{\mathrm{out}}} = \frac{\dot{m}(\rho^v - \rho^l)}{V_{\mathrm{vessel}}(\rho^l(1-\varepsilon_{\mathrm{out}}) + \rho^v\varepsilon_{\mathrm{out}})^2} \tag{6}$$

The discharged volume has to be compensated by volume sources caused by the expansion of the phases and by evaporation. The disturbance of the volume source caused by expansion for a given disturbance of pressure decay $\delta(\mathrm{d}p/\mathrm{d}t)$ is calculated by the transfer factor k_{comp}. There is a prompt change of the phase volume in case of a change of the pressure. The factor k_{comp} is calculated by:

$$k_{\mathrm{comp}} = \frac{\partial\nu_{\mathrm{comp}}}{\partial(\mathrm{d}p/\mathrm{d}t)} = -\left(\frac{\varepsilon_{\mathrm{vessel}}}{\rho^v}\frac{\mathrm{d}\rho^v}{\mathrm{d}\rho} + \frac{(1-\varepsilon_{\mathrm{vessel}})}{\rho^l}\frac{\mathrm{d}\rho^l}{\mathrm{d}p}\right) \tag{7}$$

The compensation of the volume sources can be expressed as

$$\delta\nu_{\text{out}} + \delta\nu_{\text{eva}} + \delta\nu_{\text{comp}} = \delta\nu_{\text{out}} + w_{\text{eva}}\delta\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right) + k_{\text{comp}}\delta\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right) = 0 \quad (8)$$

This leads to the transfer function w_p, which models the transfer of a disturbance of the discharged relative volume flow rate $\delta\nu_{\text{out}}$ to a disturbance of the pressure decay $\delta(\mathrm{d}p/\mathrm{d}t)$:

$$w_p(s) = \frac{-1}{w_{\text{eva}}(s) + k_{\text{comp}}} \quad (9)$$

The transfer function w_{eva} is derived at Appendix B. The result is:

$$w_{\text{eva}}(s) = \frac{\partial \nu_{\text{eva}}}{\partial(\mathrm{d}p/\mathrm{d}t)} = \frac{\rho^l - \rho^v}{h^v - h^l}\left[\frac{1-\varepsilon_{\text{vessel}}}{\rho^v}\left(\frac{1}{\rho^l} - \frac{\mathrm{d}h_s^l}{\mathrm{d}p}\right)\frac{a^l}{s+a^l} + \frac{\varepsilon_{\text{vessel}}}{\rho^l}\left(\frac{1}{\rho^v} - \frac{\mathrm{d}h_s^v}{\mathrm{d}p}\right)\frac{a^v}{s+a^v}\right] \quad (10)$$

with

$$a^l = \frac{\alpha^l f}{c_p^l(1-\varepsilon_{\text{vessel}})\rho^l} \quad \text{and} \quad a^v = \frac{\alpha^v f}{c_p^v \varepsilon_{\text{vessel}}\rho^v}$$

It describes the transfer of a disturbance of the pressure decay to the volume source caused by evaporation related on the vessel volume.

The transfer function $w_{\text{sep}}(s)$ considers the phase separation within the vessel. It was derived under the assumption of a non-inertial two-phase flow, where mass conservation equations and a drift model are the only governing equations. It reflects the disturbance of the height of the top level of the mixture resulting from the disturbance of the volume source caused by evaporation related on the vessel volume. The complete mathematical details are given in Appendix C. ν_{eva} is calculated according to Eq. (B.21).

$$w_{\text{sep}}(s) = \frac{\zeta^v}{s(1-\varepsilon_{\text{vessel}})} \times \left[\frac{1}{\tilde{\nu}}\left(\exp\left(\frac{\tilde{\nu} z_{\text{level}}}{\zeta^v}\right) - 1\right) - \frac{1}{\tilde{\nu}+s}\left(\exp\left(\frac{\tilde{\nu} z_{\text{level}}}{\zeta^v}\right) - \exp\left(-\frac{s z_{\text{level}}}{\zeta^v}\right)\right)\right] \quad (11)$$

with

$$\tilde{\nu} = \frac{\nu_{\text{eva}}}{(1-\varepsilon_{\text{vessel}})}$$

It is assumed that there is not a sharp level at top of mixture, but a linear transition from $\varepsilon_{\text{level}}$ to $\varepsilon = 1$. For the height of this transition layer, called subsequently froth, an empirical correlation from literature is used [4]:

$$z_{\text{froth}} = k_{\text{froth}}\gamma_{\text{froth}} j_{\text{level}}^v \sqrt{\frac{\rho^v}{\gamma_{\text{froth}} g(\rho^l - \rho^v)}\sqrt[4]{\frac{\gamma_{\text{froth}}}{d}}} \quad (12)$$

with

$$d = d_{\text{vessel}} \quad \text{if } d_{\text{vessel}} \leqslant d_{\text{limit}}$$
$$d = d_{\text{limit}} \quad \text{if } d_{\text{vessel}} \geqslant d_{\text{limit}}$$

$$d_{\text{limit}} = 260\gamma_{\text{froth}}\left(\frac{\rho^v}{\rho^l - \rho^v}\right)^{0.2}, \qquad \gamma_{\text{froth}} = \sqrt{\frac{\sigma}{g(\rho^l - \rho^v)}}$$

The superficial velocity of the vapour phase at the level is calculated by

$$j_{\text{level}}^v = \zeta^v \varepsilon_{\text{level}} \quad (13)$$

with $\varepsilon_{\text{level}}$ according to Eq. (C.6). The transfer coefficient k_{froth} reflects the disturbance of the vapour volume fraction at the inlet of the ventline caused by a disturbance of the location of the level. Because of the above mentioned assumption of a linear transition it is calculated by

$$k_{\text{froth}} = \frac{1-\varepsilon_{\text{level}}}{z_{\text{froth}}} \quad (14)$$

This completes the transfer of the disturbances. The feedback of an initial disturbance of the void fraction at the inlet of the ventline on itself can be modelled by these transfer functions. The transfer function for the whole systems describing the disturbance of the mass flow rate caused by a disturbance of the vapour volume fraction at the begin of the ventline is given in the Laplacian space by

$$H(s) = \frac{\delta\dot{m}}{\delta\varepsilon_{\text{in}}} = \frac{w_{\text{ventline}} k_m}{1 + H_{\text{open}}(s)} \quad (15)$$

with

$$H_{\text{open}}(s) = -w_{\text{ventline}}(k_m k_{m\nu} + k_{\varepsilon\nu}) w_p w_{\text{eva}} w_{\text{sep}} k_{\text{froth}}$$

For the analysis of stability the transfer function for the open system H_{open} is plotted for a periodic stimulation ($s = i\omega$) at the complex number plane, i.e., a Nyquist-plot is considered. The system will oscillate if the disturbance reacts in phase with an amplification greater than one. This corresponds to the Nyquist-criterion, which is well known in automation theory: The system is stable if the plot does not enclose the point (−1,0). That means if the point (−1,0) is enclosed oscillations may occur.

3. Case studies

Case studies were performed under consideration of the properties of the Pressurizer Test Facility (DHVA) at Hochschule für Technik, Wirtschaft und Sozialwesen Zittau/Görlitz (FH), which was also used for experiments described in Chapter 4 (see Table 1 for data). Typical Nyquist-plots are shown at Fig. 3. In this example case no oscillations should occur for a length of the ventline of 0.25 meter, because the point (−1,0) is not enclosed by the corresponding Nyquist-plot. In case of a ventline with a length of 0.39 meter this point is just enclosed, the amplification of a disturbance is near to 1. For case of longer ventlines the amplification increases. That means for the

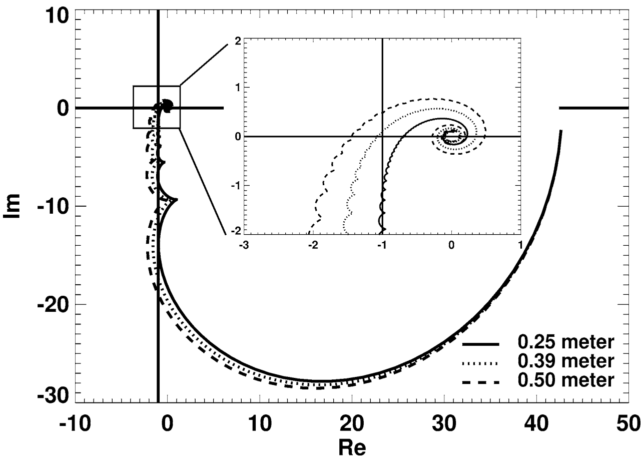

Fig. 3. Nyquist-plots for several lengths of the ventline.

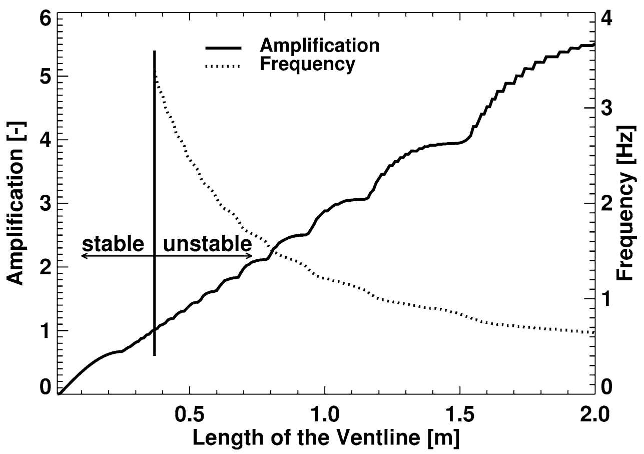

Fig. 4. Predicted amplification and frequency as a function of the ventline length.

Table 1
Reference data set

Parameter	Value
Vessel height	2.96 m
Vessel diameter	0.274 m
Ventline length	1.8 m
Ventline diameter	25 mm
Orifice diameter	6 mm
Pressure	2.25 MPa
Average volume void fraction within the ventline	90%
Velocity of the void phase	$0.7\ m \cdot s^{-1}$
Density of the interface boundary surface	$5\ m^{-1}$

example case oscillations are predicted by the linear stability analysis for a length of the ventline larger than 0.39 meter.

For the reference parameter set listed in Table 1 the calculated amplification is more than 5. That means oscillations are predicted by the stability analysis. These oscillations were also confirmed by transient calculations using the one-dimensional vessel models BLDN [5,6] and BRICK [7, 8], combined with a simple ventline model [9].

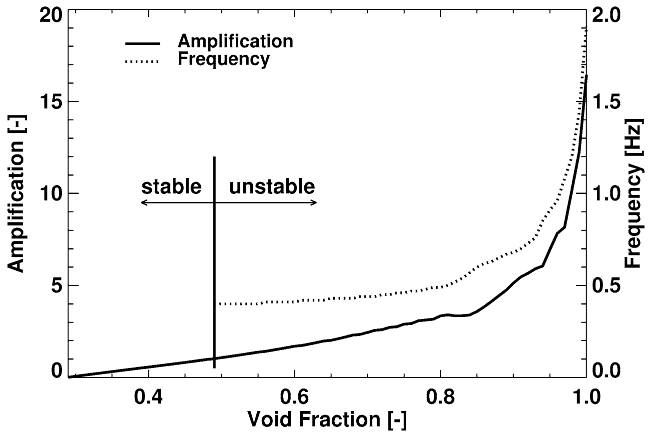

Fig. 5. Predicted amplification and frequency as a function of the discharge void fraction.

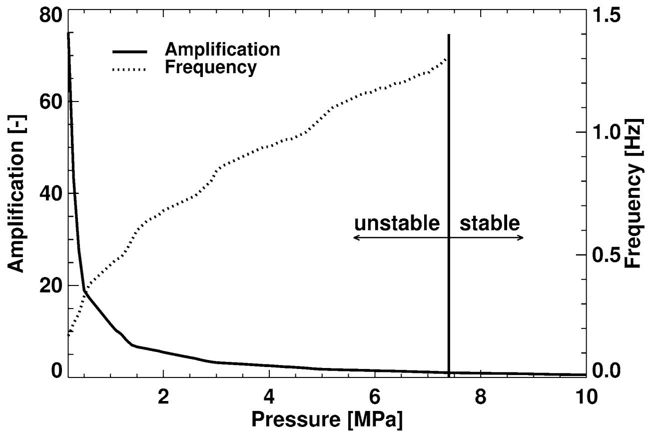

Fig. 6. Predicted amplification and frequency as a function of the pressure.

The dependency of the amplification and the frequency from the length of the ventline as important parameter is shown in Fig. 4. The calculated frequencies are valid only for very small disturbances. In practice the frequency may differ considerably amongst others because of non-linear effects. As discussed above oscillations are predicted for a length of the ventline of more than 0.39 meter.

Another important parameter is the average void fraction within the ventline. As shown in Fig. 5 the possibility of oscillations increases very strongly with the ventline void fraction. The largest amplification is observed for the extreme case of $\varepsilon \to 1$. This region of high void fraction at the outlet is always passed through during the pressure relief process (when the level collapses). We can expect the system to be stable only in case of stability for $\varepsilon \to 1$, i.e., this is a adequate condition.

In Figs. 6 and 7 the variation of the system pressure and the diameter of the orifice is shown. The possibility that oscillations occur decreases with increasing pressure and increasing diameter of the orifice. A dependency of the amplification on the vessel diameter occurs only for vessel diameter of less than 0.25 meter (see Fig. 8). That means, the tall vessel of the experimental setup (see below), which is

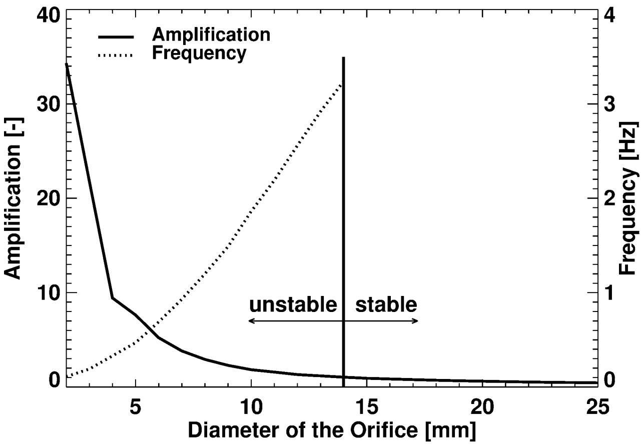

Fig. 7. Predicted amplification and frequency as a function of the orifice diameter.

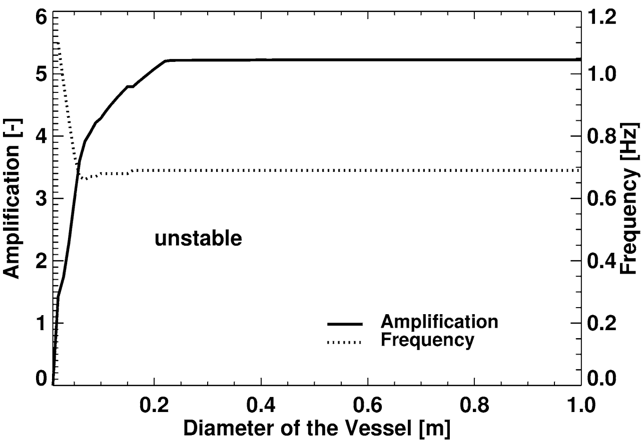

Fig. 8. Predicted amplification and frequency as a function of the vessel diameter.

also used at the reference parameter set is not a condition for the occurrence of the instabilities. The instabilities should also occur at vessels with a larger diameter.

4. Experiments and transient simulation

Experiments were carried out with water/steam at the Pressurizer Test Facility at Hochschule für Technik, Wirtschaft und Sozialwesen Zittau/Görlitz (FH). The pressure vessel is equipped with a 1.8 m long ventline with an inner diameter of 25 mm. The diameter of the relief device amounts to 6 mm. The predicted pulsations were confirmed by these experiments. Even the sound produced by the discharge has shown periodical changes, which could be clearly heard. Oscillations were found in the readings of the pressure drop over the ventline as well as in the signals of the needle shaped conductivity probes that measure the local void fraction.

The oscillations are measured already at the inlet of the ventline, meaning that they are not caused by establishing of a plug flow within the ventline. There is a time delay of

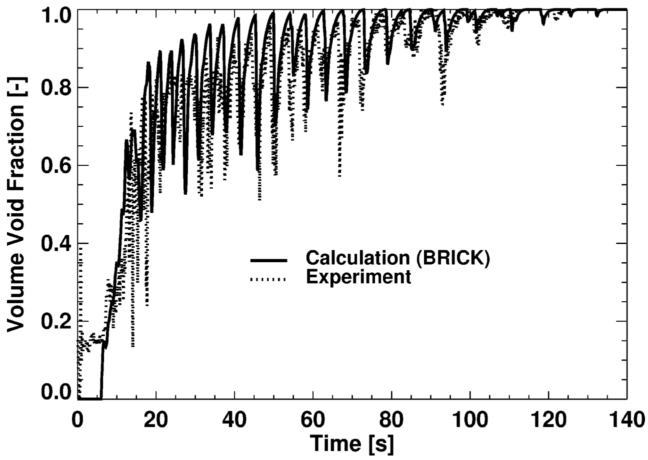

Fig. 9. Measured and calculated oscillations of the volume void fraction within the ventline.

the oscillations at the three different needle probes in the ventline, which reflects the transport time.

Transient simulations were made using the BRICK code [7,8]. Fig. 9 includes a comparison of the averaged void fraction within the ventline measured by the pressure drop and calculated by the brick code. Apart from the disagreement in the first seconds, which may be caused by non-condensibles in the experimental setup, the frequency of the oscillations as well as the sawtooth type shape and the decrease of the frequency with time show a very good agreement.

5. Conclusions

Oscillations of the mass quality in the discharged two-phase mixture connected with considerable pulsations of the discharge mass flow rate were predicted by linear stability analysis and by transient calculations. They were confirmed qualitatively by first experiments. For further investigations a more detailed modelling of froth layer, of the fluid entrainment and the ventline as well as some modifications of the experimental setup are necessary.

The probability for the occurrence of the pulsations rises with the volume of the ventline between vessel and relief device. It could be possible that these pulsations lead to additional mechanical loads on the blow down system and that they influence the efficiency of separators or condensers. In addition to the mechanism of the oscillations considered here, also other mechanisms are possible, e.g., a spontaneous transition to a slug flow regime within the ventline.

Appendix A. Critical discharge model

In the calculations, an isentropic homogeneous flow model was used. It is based on the assumption of a perfect absence of both evaporation and slip, i.e., the liquid phase is entering into the region of metastability, while the moment of both phases are in equilibrium. The following system of equations must be integrated along the flow path in the discharge channel:

$$\mathrm{d}\Delta h = v\,\mathrm{d}p \tag{A.1}$$

$$\zeta = \sqrt{2\Delta h} \tag{A.2}$$

$$\frac{\dot m}{A} = \frac{\zeta}{v} \tag{A.3}$$

Here, Eqs. (A.1) and (A.2) represent the momentum conservation, Δh is the decrease of the specific enthalpy due to the acceleration of the fluid. Eq. (A.3) describes the increase of the velocity with decreasing cross section of the discharge channel.

The assumption of a maximal thermodynamic non-equilibrium leads to a corresponding state equation. It was assumed that the density of the liquid phase remains constant during the expansion, i.e., the specific volume of the liquid preserves the initial value, which is given by saturation conditions in front of the discharge orifice, i.e., at the pressure before the start of the acceleration. This is a reasonable approximation for incompressible liquids, like water. For the state change of the vapour phase a constant adiabatic coefficient ($\kappa = 1.3$) was assumed:

$$v = v^l_{\mathrm{out}}(1 - x_{\mathrm{out}}) + v^v_{\mathrm{out}}\left(\frac{p_{\mathrm{out}}}{p}\right)^{1/\kappa} x_{\mathrm{out}} \tag{A.4}$$

After integrating Eq. (A.1) and considering Eqs. (A.2) to (A.4) we get the mass flow density as a function of the local pressure in the discharge channel:

$$\frac{\dot m}{A} = \frac{\sqrt{2\frac{p_{\mathrm{out}}}{v_{\mathrm{out}}}\left(\tilde v^l_{\mathrm{out}}(1-x_{\mathrm{out}})(1-\tilde p) + \tilde v^v_{\mathrm{out}}\frac{\kappa}{\kappa-1}(1-\tilde p^{(\kappa-1)/\kappa})x_{\mathrm{out}}\right)}}{\tilde v^l_{\mathrm{out}}(1-x_{\mathrm{out}}) + \tilde v^v_{\mathrm{out}}\tilde p^{-1/\kappa}x_{\mathrm{out}}} \tag{A.5}$$

In Eq. (A.5) related specific volumes

$$\tilde v^l_{\mathrm{out}} = \frac{v^l_{\mathrm{out}}}{v_{\mathrm{out}}} \quad \text{and} \quad \tilde v^v_{\mathrm{out}} = \frac{v^v_{\mathrm{out}}}{v_{\mathrm{out}}}$$
$$v_{\mathrm{out}} = v^l_{\mathrm{out}}(1 - x_{\mathrm{out}}) + v^v_{\mathrm{out}}x_{\mathrm{out}} \tag{A.6}$$

and a pressure ratio

$$\tilde p = \frac{p}{p_{\mathrm{out}}} \tag{A.7}$$

were introduced. The vapour mass fraction x_{out} at the inlet of the discharge channel is substituted by the void fraction $\varepsilon_{\mathrm{out}}$. In this way, the mass flow density can be made independent of the physical properties v^l_{out} and v^v_{out}:

$$\frac{\dot m}{A} = \frac{\sqrt{2\frac{p_{\mathrm{out}}}{v_{\mathrm{out}}}\left((1-\varepsilon_{\mathrm{out}})(1-\tilde p) + \varepsilon_{\mathrm{out}}\frac{\kappa}{\kappa-1}\left(1-\tilde p^{(\kappa-1)/\kappa}\right)\right)}}{(1-\varepsilon_{\mathrm{out}}) + \varepsilon_{\mathrm{out}}\tilde p^{-1/\kappa}} \tag{A.8}$$

The only physical property remaining in Eq. (A.8) is the adiabatic coefficient for the void. The critical pressure ratio is reached, when the mass flow density $\dot m/A$ is maximal, i.e., when the corresponding cross Appendix A is minimal. The maximum of Eq. (A.8) is found numerically with the initial void volume fraction as a parameter. The results were fitted by the following correlation:

$$\dot m = A_{\mathrm{crit}}\sqrt{\frac{p_{\mathrm{out}}}{v_{\mathrm{out}}}}\left(\sqrt{2} - 1.0484\varepsilon^{0.438}_{\mathrm{out}} + 0.30603\varepsilon_{\mathrm{out}}\right) \tag{A.9}$$

Considering the relation

$$\frac{1}{v_{\mathrm{out}}} = \rho^l(1 - \varepsilon_{\mathrm{out}}) + \rho^v\varepsilon_{\mathrm{out}} \tag{A.10}$$

and introducing a factor μ, which considers the contraction of the fluid ($\mu = A_{\mathrm{crit}}/A_{\mathrm{orif}}$), finally yields:

$$\dot m = \mu\phi A_{\mathrm{orif}}\sqrt{p\left(\rho^l(1-\varepsilon_{\mathrm{out}}) + \rho^v\varepsilon_{\mathrm{out}}\right)} \tag{A.11}$$

with

$$\phi = \sqrt{2} - 1.0484\varepsilon^{0.438}_{\mathrm{out}} + 0.30603\varepsilon_{\mathrm{out}}$$

For the discharge model $\varepsilon_{\mathrm{out}}$ is an input parameter characterizing the state of the two-phase mixture in front of the discharge device. In case of a single phase flow ($\varepsilon_{\mathrm{out}} = 0$), the mass flow density reduces to the value for liquid discharge according to the Bernoulli-equation. At growing $\varepsilon_{\mathrm{out}}$ the mass flow rate decreases. Main advantage of Eq. (A.11) is that it is very easy to handle. In reality, the metastability is more or less destroyed due to evaporation, which leads to an overestimation of the mass flow by Eq. (A.11). The accuracy of the model increases with decreasing length of the discharge channel.

Appendix B. Transfer function for the rate of evaporation

Starting point is the conservation equation for the specific energy of the phase i:

$$\frac{\partial}{\partial t}\varepsilon^i\rho^i u^i + \frac{\partial}{\partial z}\varepsilon^i\rho^i h^i\zeta^i + p\frac{\partial}{\partial t}\varepsilon^i$$
$$= \pm\Gamma h^i_s + q^i_{\mathrm{phases}} + q^i_{\mathrm{sources}} \tag{B.1}$$

with

$$\Gamma = -\frac{q^i_{\mathrm{phases}} + q^j_{\mathrm{phases}}}{r}$$

$$q^i_{\mathrm{phases}} = \alpha^i f\left(T^i_s - T^i\right) \approx \frac{\alpha^i f}{c^i_p}\left(h^i_s - h^i\right)$$

Γ is the mass transfer rate from liquid to void phase per volume. The positive sign is valid for the vapour phase, the negative for the liquid. q^i_{phases} is the heat flow density from the phase i to the boundary surface of the phase and q^i_{sources} is the heat flow density from the vessel wall or chemical reaction to phase i.

Substitution

$$u^i = h^i - \frac{p}{\rho^i} \tag{B.2}$$

leads to

$$\frac{\partial}{\partial t}\varepsilon^i\rho^i h^i + \frac{\partial}{\partial z}\varepsilon^i\rho^i h^i\zeta^i - \varepsilon^i\frac{\mathrm{d}p}{\mathrm{d}t} = \pm\Gamma h_s^i + q^i_{\text{sources}} + q^i_{\text{phases}} \tag{B.3}$$

Considering the conservation equation for the mass

$$\frac{\partial}{\partial t}\varepsilon^i\rho^i + \frac{\partial}{\partial z}\varepsilon^i\rho^i\zeta^i = \pm\Gamma \tag{B.4}$$

and the transition to the total differential

$$\frac{\partial}{\partial t}h^i + \zeta^i\frac{\partial}{\partial z}h^i = \frac{\mathrm{d}}{\mathrm{d}t}h^i \tag{B.5}$$

one gets

$$\frac{\mathrm{d}}{\mathrm{d}t}h^i = \frac{1}{\rho^i}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{\pm\Gamma(h_s^i - h^i) + \dot{q}^i_{\text{sources}} + q^i_{\text{phases}}}{\varepsilon^i\rho^i} \tag{B.6}$$

Replacement of G and q^i_{phases} according the terms given at Eq. (B.1) yields:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}h^i &= \frac{1}{\rho^i}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{q_{\text{sources}}}{\varepsilon^i\rho^i} \\ &\quad + \left(1 \mp \frac{h_s^i - h^i}{r}\right)\frac{\alpha^i f}{c_p^i\varepsilon^i\rho^i}\left(h_s^i - h^i\right) \\ &\quad \mp \frac{h_s^i - h^i}{r}\frac{\alpha^j f}{c_p^j\varepsilon^i\rho^i}\left(h_s^j - h^j\right)\end{aligned} \tag{B.7}$$

where the upper sign is valid for the vapour phase and the lower for the liquid phase. The coupled system of differential equations is decoupled by the assumptions that there occurs only a small deviation from the equilibrium state and that the system is away from the thermodynamic critical state. For these assumptions is

$$\Delta h^i = h^i - h_s \ll r \tag{B.8}$$

and terms of second order can be neglected. In the end one gets

$$\frac{\mathrm{d}}{\mathrm{d}t}\Delta h^i + \frac{\alpha^i f}{c_p^i\varepsilon^i\rho^i}\Delta h^i = \frac{1}{\rho^i}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{q_{\text{sources}}}{\varepsilon^i p^i} - \frac{\mathrm{d}}{\mathrm{d}t}h_s^i \tag{B.9}$$

This equation is used to consider the time dependent behaviour of Δh_i in case of a sudden change of $\mathrm{d}p/\mathrm{d}t$ at $t = 0$ (jump from $\mathrm{d}p/\mathrm{d}t$ to $\mathrm{d}p/\mathrm{d}t + \delta\{\mathrm{d}p/\mathrm{d}t\}$. Defining

$$a^i = \frac{\alpha^i f}{c_p^i\varepsilon^i\rho^i} \tag{B.10}$$

and

$$\begin{aligned}b^i &= \frac{1}{\rho^i}\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{q_{\text{sources}}}{\varepsilon^i\rho^i} - \frac{\mathrm{d}}{\mathrm{d}t}h_s^i \\ &= \frac{q_{\text{sources}}}{\varepsilon^i\rho^i} + \left(\frac{1}{\rho^i} - \frac{\mathrm{d}}{\mathrm{d}p}h_s^i\right)\frac{\mathrm{d}p}{\mathrm{d}t}\end{aligned} \tag{B.11}$$

for the values at the working point and

$$\delta b^i = \left(\frac{1}{\rho^i} - \frac{\mathrm{d}}{\mathrm{d}p}h_s^i\right)\delta\left\{\frac{\mathrm{d}p}{\mathrm{d}t}\right\} \tag{B.12}$$

the differential equation has the general solution for $t > 0$

$$\Delta h^i = C^i\exp\left(-a^i t\right) + \frac{b^i + \delta b^i}{a^i} \tag{B.13}$$

With the conditions

$$\Delta h^i(t=0) = \frac{b^i}{a^i} \quad \text{and} \quad \lim_{t\to\infty}\Delta h^i(t) = \frac{b^i + \delta b^i}{a^i} \tag{B.14}$$

the special solution for the considered jump function for $\mathrm{d}p/\mathrm{d}t$ is found as

$$\begin{aligned}\Delta h^i &= \Delta h_0^i + \delta\left(\Delta h^i\right) \\ &= \frac{b^i}{a^i} + \left(1 - \exp\left(-a^i t\right)\right)\frac{\delta b^i}{a^i}\end{aligned} \tag{B.15}$$

The volume source density caused by evaporation is calculated by

$$\begin{aligned}\nu_{\text{eva}} &= \left(\frac{1}{\rho^v} - \frac{1}{\rho^l}\right)\Gamma \\ &= \frac{1}{r}\left(\frac{1}{\rho^v} - \frac{1}{\rho^l}\right)\left(\frac{\alpha^l f}{c_p^l}\Delta h^l + \frac{\alpha^v f}{c_p^v}\Delta h^v\right)\end{aligned} \tag{B.16}$$

Considering Eqs. (B.10), (B.11) and (B.12) the disturbance is given by

$$\delta\nu_{\text{eva}} = W_{\text{eva}}(t)\delta\left\{\frac{\mathrm{d}p}{\mathrm{d}t}\right\} \tag{B.17}$$

with

$$\begin{aligned}W_{\text{eva}} &= \frac{1}{r}\left(\frac{1}{\rho^v} - \frac{1}{\rho^l}\right)\left(\varepsilon^l\rho^l\left(\frac{1}{\rho^l} - \frac{\mathrm{d}}{\mathrm{d}p}h_s^l\right)\left(1 - \exp\left(-a^l t\right)\right)\right. \\ &\quad \left. + \varepsilon^v\rho^v\left(\frac{1}{\rho^v} - \frac{\mathrm{d}}{\mathrm{d}p}h_s^v\right)\left(1 - \exp\left(-a^v t\right)\right)\right)\end{aligned}$$

Using the relations for the Laplace transformation

$$L\{1\} = \frac{1}{s} \quad \text{and} \quad L\left\{\exp\left(-a^i t\right)\right\} = \frac{1}{a^i + s} \tag{B.18}$$

and considering the whole vessel volume $\varepsilon^v \to \varepsilon_{\text{vessel}}$ the transfer function is found as

$$\begin{aligned}w_{\text{eva}}(s) &= \frac{L\{W_{\text{eva}}(t)\}}{L\{1\}} \\ &= \frac{\rho^l - \rho^v}{h^v - h^l}\left[\frac{1 - \bar{\varepsilon}}{\rho^v}\left(\frac{1}{\rho^l} - \frac{\mathrm{d}h_s^l}{\mathrm{d}p}\right)\frac{a^l}{s + a^l}\right. \\ &\quad \left. + \frac{\bar{\varepsilon}}{\rho^l}\left(\frac{1}{\rho^v} - \frac{\mathrm{d}h_s^v}{\mathrm{d}p}\right)\frac{a^v}{s + a^v}\right]\end{aligned} \tag{B.19}$$

The special case of thermodynamic equilibrium is obtained by the limit test $a^i \to \infty$, what yields

$$\begin{aligned}A_{\text{eva}} &= \lim_{a^i\to\infty} w_{\text{eva}}(s) \\ &= \frac{\rho^l - \rho^v}{h^v - h^l}\left[\frac{1}{\rho^v\rho^l} - \frac{1 - \bar{\varepsilon}}{\rho^v}\frac{\mathrm{d}h_s^l}{\mathrm{d}p} - \frac{\bar{\varepsilon}}{\rho^l}\frac{\mathrm{d}h_s^v}{\mathrm{d}p}\right]\end{aligned} \tag{B.20}$$

This transfer factor does not depend on s. The evaporation rate change prompt with a change of $\mathrm{d}p/\mathrm{d}t$.

The volume source caused by evaporation related on the vessel volume for the working point of the system is calculated according to

$$\nu_{\text{eva}} = A_{\text{eva}} \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{A_{\text{eva}}\dot{m}}{(A_{\text{eva}} + A_{\text{comp}}) V_{\text{Vessel}} (\rho_L (1-\varepsilon_{\text{out}}) + \rho_V \varepsilon_{\text{out}})} \tag{B.21}$$

Appendix C. Transfer function for the phase separation

The change of a vapour volume with the height Δz in the time Δt is given by:

$$\Delta V^v = \dot{V}^v(z,t)\Delta t - \dot{V}^v(z+\Delta z, t)\Delta t + \nu^v(z,t) A_{\text{vessel}} \Delta z \Delta t \tag{C.1}$$

Dividing this equation by $A_{\text{vessel}}\Delta z \Delta t$ and considering

$$j^v(z,t) = \frac{\dot{V}^v(z,t)}{A_{\text{vessel}}} = \zeta^v \varepsilon(z,t) \tag{C.2}$$

one gets

$$\frac{\partial \varepsilon}{\partial t} + \zeta^v \frac{\partial \varepsilon}{\partial z} = \nu^v(z,t) \tag{C.3}$$

The liquid velocity is assumed to be zero and the vapour velocity ζ^v is assumed to be constant in space and time in order to simplify the derivation. The vapour volume source is caused by evaporation. Volume sources caused by the compressibility of the phases are neglected for this derivation. For this reason the vertical distribution of the volume source only depends on the volume fraction of the liquid phase:

$$\nu^v(z,t) = \bar{\nu}^v \frac{1-\varepsilon(z,t)}{1-\varepsilon_{\text{vessel}}} \tag{C.4}$$

The disturbance of the location of the top level of the mixture mainly depends on the sum of the changes of volume sources for all phases. For this reason the averaged volume source for the vapour phase $\bar{\nu}^v$ is replaced by ν_{eva} (see Eq. (B.21)), which is the sum of the volume sources caused by evaporation for both phases. The resulting differential equation is

$$\frac{\partial \varepsilon}{\partial t} + \zeta^v \frac{\partial \varepsilon}{\partial z} = \tilde{\nu}(1-\varepsilon) \quad \text{with } \tilde{\nu} = \frac{\nu_{\text{eva}}}{1-\varepsilon_{\text{vessel}}} \tag{C.5}$$

For the working point ν_{eva} is not a function of time. For this reason is an appropriate solution of this equation.

$$\varepsilon = 1 - \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right) \tag{C.6}$$

A small disturbance of ν causes a disturbance of ε. This leads to:

$$\frac{\partial}{\partial t}(\varepsilon + \delta\varepsilon) + \zeta^v \frac{\partial}{\partial z}(\varepsilon + \delta\varepsilon) = (\tilde{\nu} + \delta\tilde{\nu})\big(1 - (\varepsilon + \delta\varepsilon)\big) \tag{C.7}$$

Neglecting terms of second order and considering Eq. (C.5) an equation for the disturbance is received:

$$\frac{\partial}{\partial t}\delta\varepsilon + \zeta^v \frac{\partial}{\partial z}\delta\varepsilon + \tilde{\nu}\delta\varepsilon = (1-\varepsilon)\delta\tilde{\nu} = \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right)\delta\tilde{\nu} \tag{C.8}$$

The general solution of this partial differential equation is

$$\delta\varepsilon = C \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right) F\left(t - \frac{z}{\zeta^v}\right) + t \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right)\delta\tilde{\nu} \tag{C.9}$$

where F is any function. Considering the conditions

$$\delta\varepsilon(t=0, z) = 0 \tag{C.10}$$

and

$$\delta\varepsilon(t \to \infty, z) = \frac{z}{\zeta^v} \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right)\delta\tilde{\nu} \tag{C.11}$$

a special solution is found as

$$\begin{aligned} \delta\varepsilon &= t \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right)\delta\tilde{\nu} \qquad \text{for } t \leqslant \frac{z}{\zeta^v} \\ \delta\varepsilon &= \frac{z}{\zeta^v} \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right)\delta\tilde{\nu} \quad \text{for } t \geqslant \frac{z}{\zeta^v} \end{aligned} \tag{C.12}$$

Using the jump function Θ this can be written as

$$\delta\varepsilon = \left[t - \left(t - \frac{z}{\zeta^v}\right)\Theta\left(t - \frac{z}{\zeta^v}\right)\right] \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right)\delta\tilde{\nu} \tag{C.13}$$

The Laplace transformation yields:

$$L\{\delta\varepsilon\} = \frac{1 - \exp(-zs/\zeta^v)}{s^2} \exp\left(-\frac{\tilde{\nu} z}{\zeta^v}\right)\delta\tilde{\nu} \tag{C.14}$$

Assuming the conservation of the liquid volume the displacement of the level is given by:

$$\delta z_{\text{level}} = \frac{\int_0^{z_{\text{level}}} \delta\varepsilon \, \mathrm{d}z}{(1-\varepsilon_{\text{level}})} \tag{C.15}$$

That means the searched transfer function has to be calculated according to:

$$w_{\text{sep}} = \frac{\delta z_{\text{level}}}{\delta \nu_{\text{eva}}} = \frac{1}{(1-\varepsilon_{\text{level}})} \frac{\int_0^{z_{\text{level}}} L\{\delta\varepsilon\}\,\mathrm{d}z}{\delta\nu_{\text{eva}} L\{1\}} \tag{C.16}$$

According to Eq. (C.6) applies

$$\frac{1}{(1-\varepsilon_{\text{level}})} = \exp\left(\frac{\tilde{\nu} z_{\text{level}}}{\zeta^v}\right) \tag{C.17}$$

Execution of the integration and consideration of Eq. (C.5), yields

$$w_{\text{sep}}(s) = \frac{\zeta^v}{s(1-\varepsilon_{\text{vessel}})} \times \left[\frac{1}{\tilde{\nu}}\left(\exp\left(\frac{\tilde{\nu} z_{\text{level}}}{\zeta^v}\right) - 1\right) - \frac{1}{\tilde{\nu} + s}\left(\exp\left(\frac{\tilde{\nu} z_{\text{level}}}{\zeta^v}\right) - \exp\left(-\frac{s z_{\text{level}}}{\zeta^v}\right)\right)\right]$$

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